

Strange Matter in Rotating Compact Stars

Debades Bandyopadhyay

*Saha Institute of Nuclear Physics,
Kolkata, India*

Collaborators: *Sarmistha Banik (Kolkata)*
Matthias Hanauske (Frankfurt)

Neutron Stars are one of the **densest form of matter** in the observable universe.

Neutron star matter is **cold and highly dense**. The matter density in the core exceeds by **a few times** normal nuclear matter density.

*Observations of **binary pulsars** and **isolated neutron stars** provide informations about **measured masses, radii and pulsar periods**.*

The theoretical **mass-radius relationships** of compact stars are directly compared with **measured masses and radii**.

*Consequently, the **composition** and **EoS** of dense matter in a **neutron star interior** might be probed.*

Exotica in Neutron Star Interior

Various *strangeness-rich* components of matter such as hyperons, Bose-Einstein Condensates of kaons & quarks, may appear in neutron stars.

Hyperons

- Hyperons produced at the cost of the nucleons.



- Chemical equilibrium through weak processes,

$$p + e^- \rightarrow \Lambda + \nu_e, \quad n + e^- \rightarrow \Xi^- + \nu_e$$

$$\mu_i = b_i \mu_n - q_i \mu_e$$

- Threshold Condition for Hyperons

$$\mu_n - q_i \mu_e \geq m_B^* + g_{\omega B} \omega_0 + g_{\rho B} \rho_0 \tau_3$$

Bose-Einstein condensates

The processes responsible for *p*-wave pion condensate/*s*-wave kaon condensate in compact stars,



- Threshold conditions for Bose-Einstein condensation of mesons

For K^- $\omega_{K^-} = \mu_e$.

For π^- $\omega_{\pi^-} = \mu_e$.

Ref: S. Banik, D. Bandyopadhyay, Phys.Rev.C63 (2001) 035802

S. Banik, D. Bandyopadhyay, Phys.Rev.C64 (2001) 055805

S. Banik, D. Bandyopadhyay, Phys.Rev.C66 (2002) 065801

Quark Matter

Strange matter containing u , d , s quarks may be the ground state of matter (energy/baryon < 930 MeV at finite density).

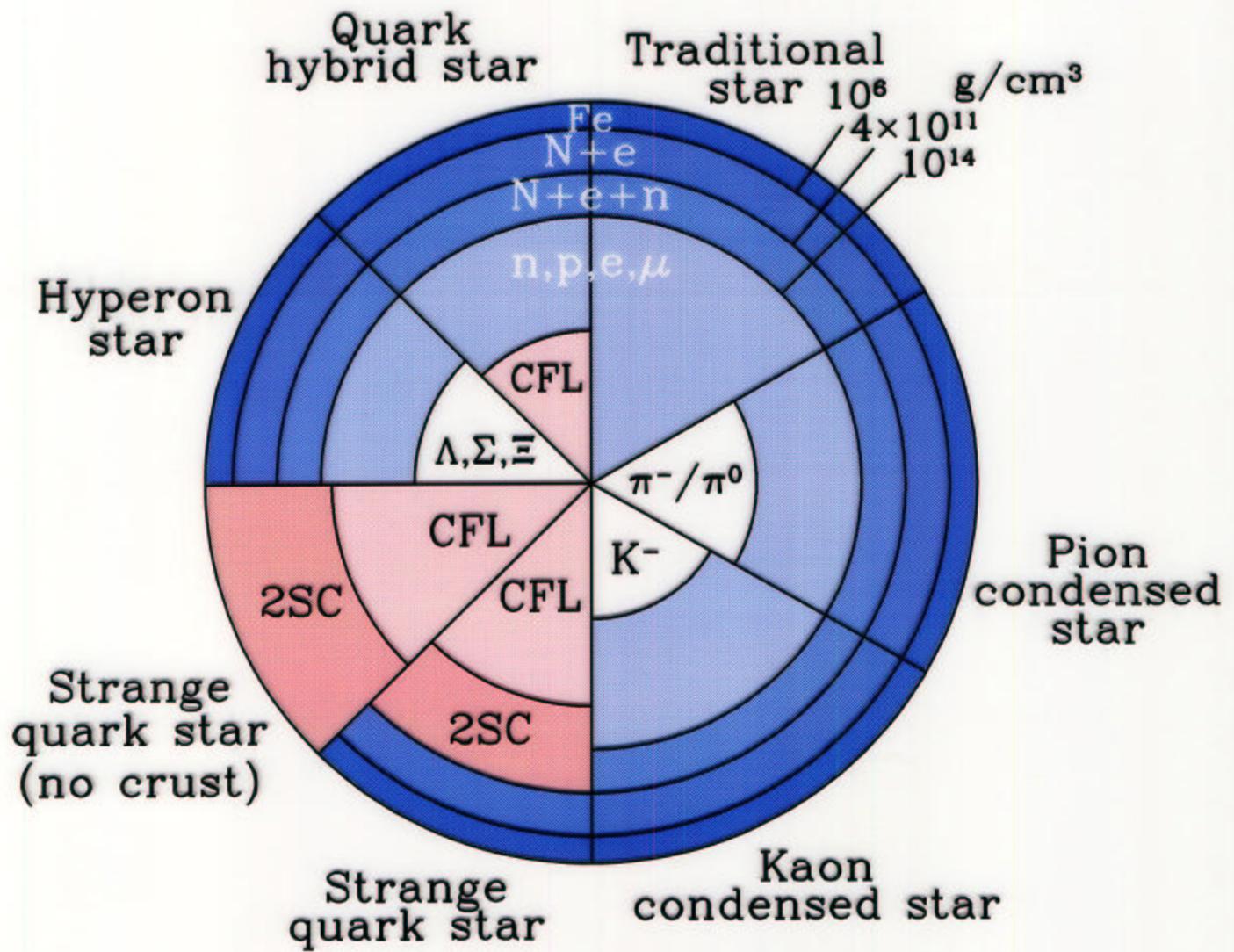
Ref: E. Witten, Phys. Rev. D30 (1984) 272

Quarks are in chemical equilibrium:

$$d \rightarrow u + e^- + \bar{\nu}_e, \quad s \rightarrow u + e^- + \bar{\nu}_e;$$
$$\mu_d = \mu_u + \mu_e, \quad \mu_s = \mu_d$$

MIT Bag model: $P \rightarrow P - B$, & $\epsilon \rightarrow \epsilon + B$

Recently it has been predicted that quark matter might be a color superconductor. Quarks near their Fermi surfaces form Cooper pairs due to the attractive quark-quark interaction in color antisymmetric channel.



Equilibrium Structures of Neutron Stars

Einstein's field equations,

$$G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R = 8\pi T^{\mu\nu}$$

- Metric tensor $g^{\mu\nu}$ for non-rotating and rotating stars,
- EoS from the stellar matter Lagrangian \mathcal{L} .

We consider **two cases**,

- First order phase transition from hadron to \bar{K} condensed matter (**HK**),
- Both first order K^- condensation and quark deconfinement (**HKQ**).

THE HADRONIC PHASE

$$\begin{aligned}
\mathcal{L} = & \sum_B \bar{\psi}_B (i\gamma_\mu \partial^\mu - m_B + g_{\sigma B} \sigma - g_{\omega B} \gamma_\mu \omega^\mu \\
& - \frac{1}{2} g_{\rho B} \gamma_\mu \vec{\tau}_B \cdot \vec{\rho}^\mu) \psi_B \\
& + \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) - U(\sigma) \\
& - \frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu \\
& - \frac{1}{4} \rho_{\mu\nu} \cdot \rho^{\mu\nu} + \frac{1}{2} m_\rho^2 \rho_\mu \cdot \rho^\mu + \mathcal{L}_{YY} \\
& + \sum_{e^-, \mu^-} \bar{\psi}_\lambda (i\gamma_\mu \partial^\mu - m) \psi_\lambda.
\end{aligned}$$

where, $U(\sigma) = \frac{1}{3} b m_N (g_{\sigma N} \sigma)^3 + \frac{1}{4} c (g_{\sigma N} \sigma)^4$.

Hyperon-Hyperon interaction:

$$\begin{aligned}
\mathcal{L}_{YY} = & \sum_B \bar{\psi}_B (g_{\sigma^* B} \sigma^* - g_{\phi B} \gamma_\mu \phi^\mu) \psi_B \\
& + \frac{1}{2} (\partial_\mu \sigma^* \partial^\mu \sigma^* - m_{\sigma^*}^2 \sigma^{*2}) \\
& - \frac{1}{4} \phi_{\mu\nu} \phi^{\mu\nu} + \frac{1}{2} m_\phi^2 \phi_\mu \phi^\mu .
\end{aligned}$$

THE ANTIKAON CONDENSED PHASE

- *Composition:* Baryons, antikaon condensates and leptons,
- Baryons *embedded in the condensates.*

The Lagrangian density for (anti)kaons in the minimal coupling scheme

$$\mathcal{L}_K = D_\mu^* \bar{K} D^\mu K - m_K^{*2} \bar{K} K .$$

The covariant derivative

$$D_\mu = \partial_\mu + ig_{\omega K} \omega_\mu + ig_{\phi K} \phi_\mu + ig_{\rho K} \boldsymbol{\tau}_K \cdot \boldsymbol{\rho}_\mu .$$

The effective mass of (anti)kaons is

$$m_K^* = m_K - g_{\sigma K} \sigma - g_{\sigma^* K} \sigma^* .$$

The equation of motion for kaons

$$(D_\mu D^\mu + m_K^{*2}) K = 0$$

In the mean field approximation, the meson field equations in the presence of antikaons

$$m_\sigma^2 \sigma = -\frac{\partial U}{\partial \sigma} + \sum_B g_{\sigma B} n_B^s + g_{\sigma K} \sum_{\bar{K}} n_{\bar{K}}$$

$$m_{\sigma^*}^2 \sigma^* = \sum_B g_{\sigma^* B} n_B^s + g_{\sigma^* K} \sum_{\bar{K}} n_{\bar{K}}$$

$$m_\omega^2 \omega_0 = \sum_B g_{\omega B} n_B - g_{\omega K} \sum_{\bar{K}} n_{\bar{K}}$$

$$m_\phi^2 \phi_0 = \sum_B g_{\phi B} n_B - g_{\phi K} \sum_{\bar{K}} n_{\bar{K}}$$

$$m_\rho^2 \rho_{03} = \sum_B g_{\rho B} I_{3B} n_B + g_{\rho K} \sum_{\bar{K}} I_{3\bar{K}} n_{\bar{K}}$$

The scalar density and baryon number density

$$n_B^S = \frac{2J_B + 1}{2\pi^2} \int_0^{k_{FB}} \frac{m_B^*}{(k^2 + m_B^{*2})^{1/2}} k^2 dk ,$$

$$n_B = (2J_B + 1) \frac{k_{FB}^3}{6\pi^2} .$$

with $m_B^ = m_B - g_{\sigma B} \sigma - g_{\sigma^* B} \sigma^*$*

The total energy density, $\varepsilon = \varepsilon_B + \varepsilon_l + \varepsilon_{\bar{K}}$,

$$\begin{aligned}
\varepsilon = & \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{3}g_2\sigma^3 + \frac{1}{4}g_3\sigma^4 + \frac{1}{2}m_{\sigma^*}^2\sigma^{*2} \\
& + \frac{1}{2}m_\omega^2\omega_0^2 + \frac{1}{2}m_\phi^2\phi_0^2 + \frac{1}{2}m_\rho^2\rho_{03}^2 \\
& + \sum_B \frac{2J_B + 1}{2\pi^2} \int_0^{k_{FB}} (k^2 + m_B^{*2})^{1/2} k^2 dk \\
& + \sum_l \frac{1}{\pi^2} \int_0^{K_{Fl}} (k^2 + m_l^2)^{1/2} k^2 dk \\
& + m_K^* (n_{K^-} + n_{\bar{K}^0}) .
\end{aligned}$$

The pressure

$$\begin{aligned}
P = & -\frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4 \\
& - \frac{1}{2}m_{\sigma^*}^2\sigma^{*2} + \frac{1}{2}m_\omega^2\omega_0^2 + \frac{1}{2}m_\phi^2\phi_0^2 + \frac{1}{2}m_\rho^2\rho_{03}^2 \\
& + \frac{1}{3} \sum_B \frac{2J_B + 1}{2\pi^2} \int_0^{k_{FB}} \frac{k^4 dk}{(k^2 + m_B^{*2})^{1/2}} \\
& + \frac{1}{3} \sum_l \frac{1}{\pi^2} \int_0^{K_{Fl}} \frac{k^4 dk}{(k^2 + m_l^2)^{1/2}} .
\end{aligned}$$

The Mixed Phase

Gibbs phase rules

$$\begin{aligned} P^I &= P^{II}, \\ \mu^I &= \mu^{II}. \end{aligned}$$

Conditions of global charge neutrality

$$(1 - \chi)Q^I + \chi Q^{II} = 0,$$

Baryon number conservation

$$n_B = (1 - \chi)n^I + \chi n^{II}$$

Total energy density

$$\epsilon = (1 - \chi)\epsilon^I + \chi\epsilon^{II}.$$

Parameters of the model

- *Nucleon-meson coupling constants*

ρ_0	E/B	a_{sym}	m_n^*/m_n	K		
$0.153 fm^{-3}$	$-16.3 MeV$	$32.5 MeV$	0.70	$300 MeV$		
$g_{\sigma N}$	$g_{\omega N}$	$g_{\rho N}$	g_2	g_3		
9.5708	10.5964	8.1957	12.2817	-8.978	0	0

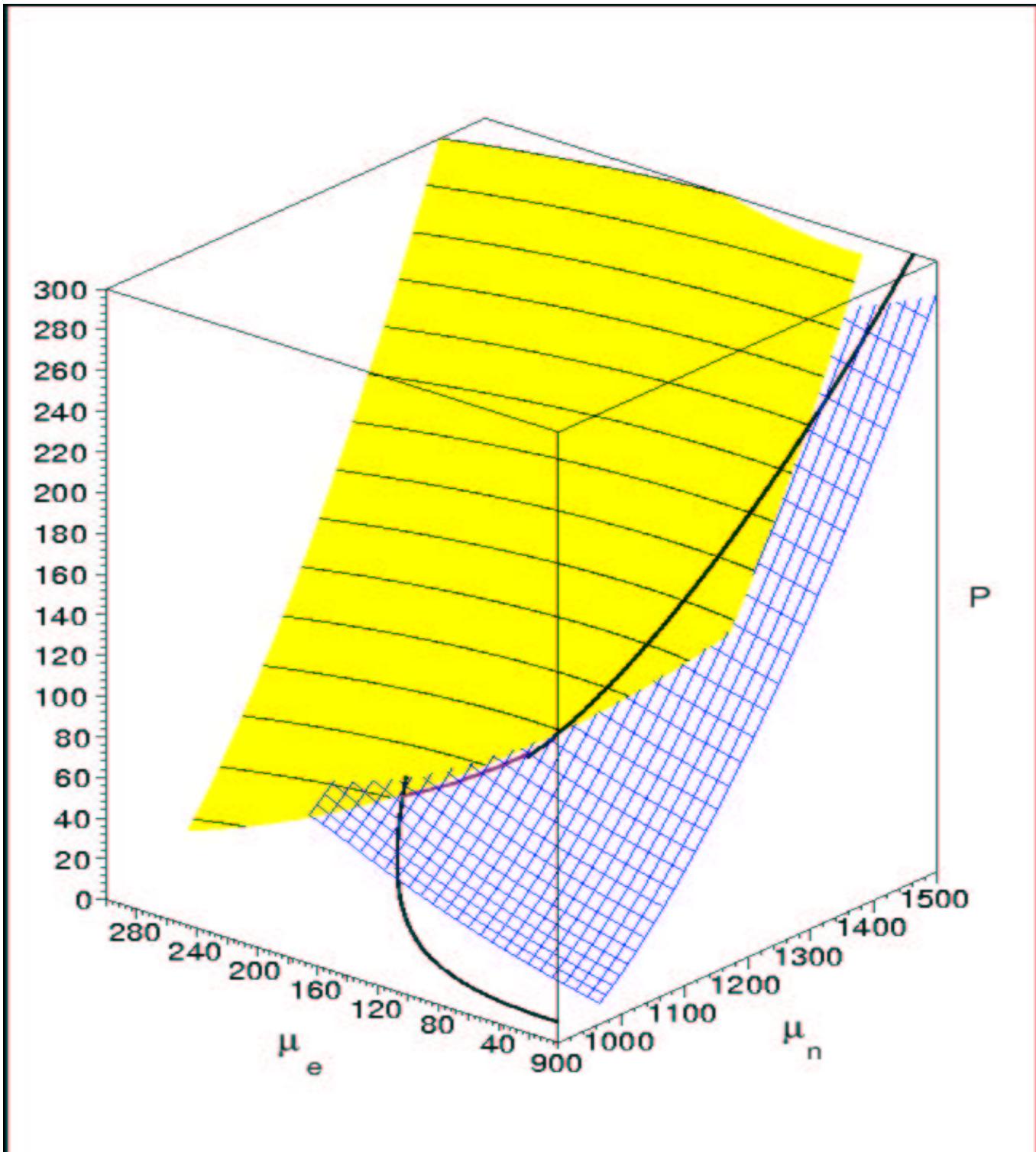
- *Hyperon-meson coupling constants* are determined using $SU(6)$ symmetry of the quark model and hypernuclei data

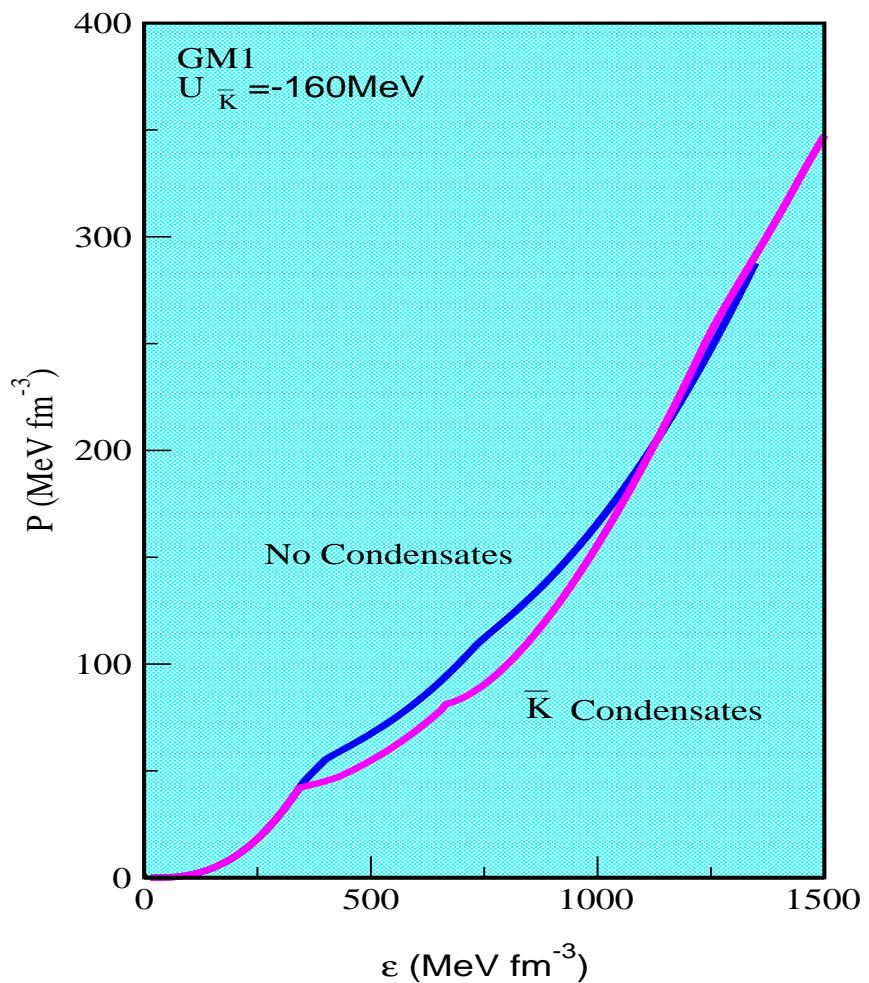
$$U_{\Lambda}^N = -30 MeV, \quad U_{\Sigma}^N = +30 MeV \quad \& \quad U_{\Xi}^N = -18 MeV$$

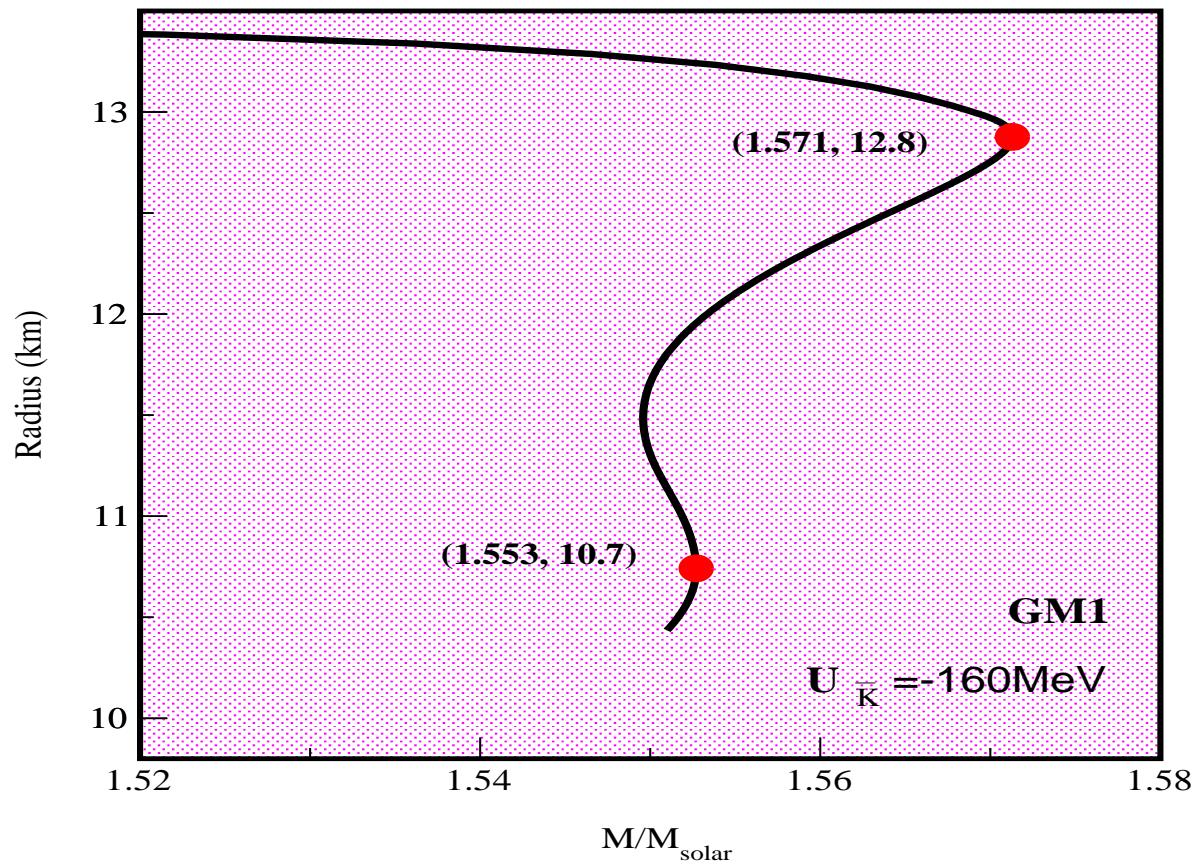
$$U_{\Xi}^{(\Xi)}(n_0) = U_{\Lambda}^{(\Xi)}(n_0) = 2U_{\Xi}^{(\Lambda)}(n_0) = 2U_{\Lambda}^{(\Lambda)}(n_0) = -40 MeV.$$

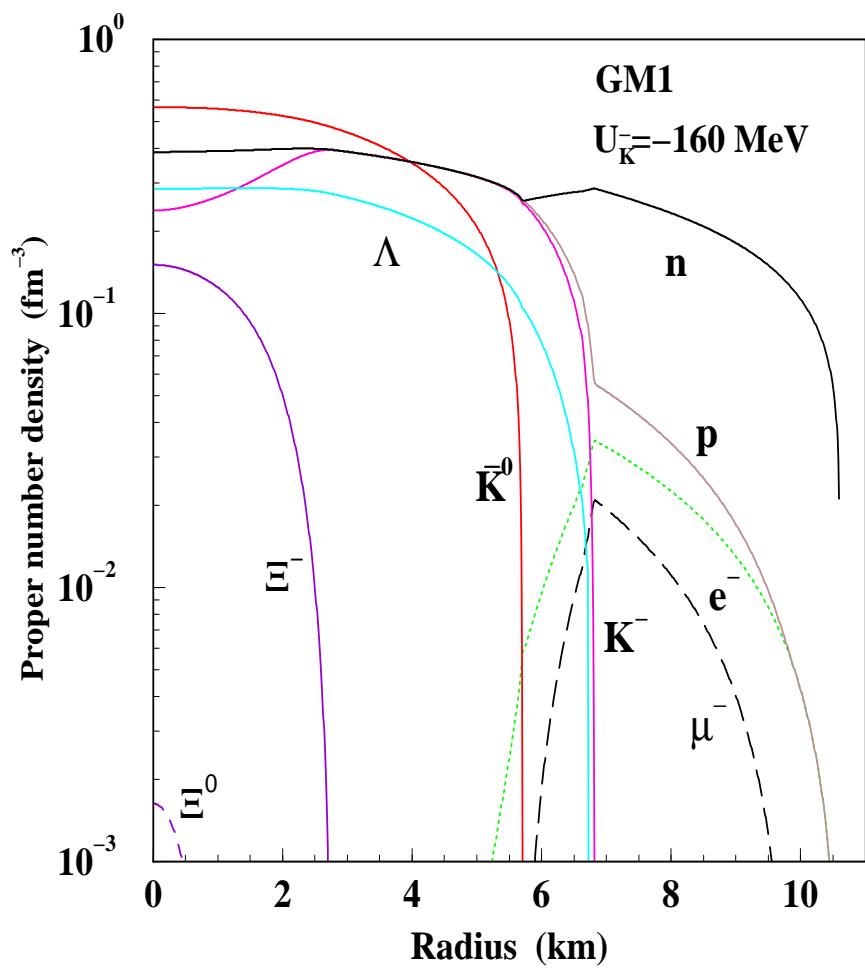
- *Kaon-meson coupling constants*
- *Strongly attractive* antikaon optical potential in nuclear matter is evident from *flow* properties of antikaons and K^- atomic data.

$$g_{\omega K} = \frac{1}{3} g_{\omega N} \quad g_{\rho K} = g_{\rho N} \cdot U_{\bar{K}}(n_0) = -160 \text{ MeV}, \\ g_{\sigma K} = 2.9937 \quad g_{\sigma^* K} = 2.65 \quad \& \quad \sqrt{2} g_{\phi K} = 6.04.$$









Structure of Rotating Compact stars

The metric of a stationary and axisymmetric rotating star,

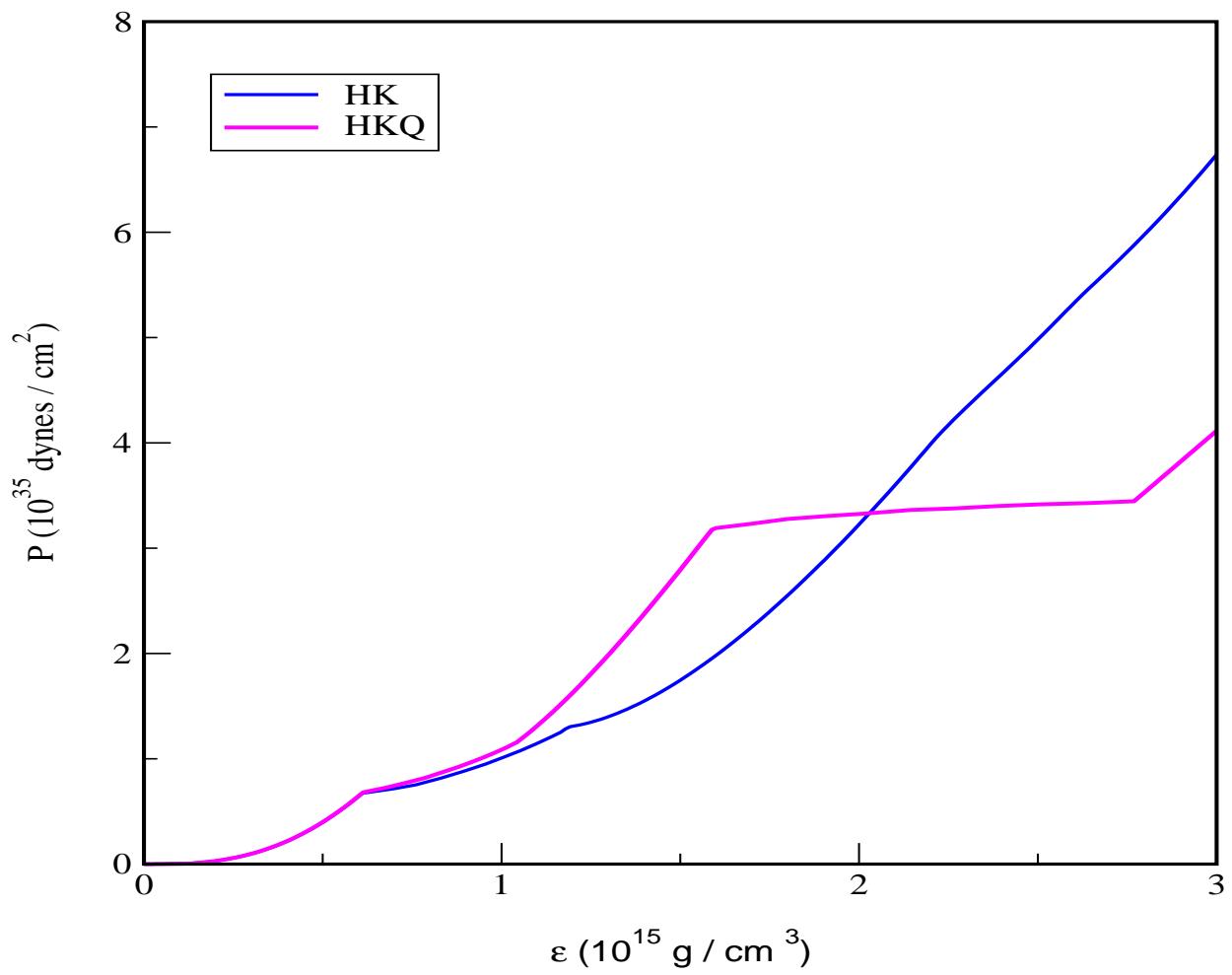
$$ds^2 = -e^{2\nu}dt^2 + e^{2\psi}(d\phi - \omega dt)^2 + e^{2\mu}d\theta^2 + e^{2\lambda}dr^2$$

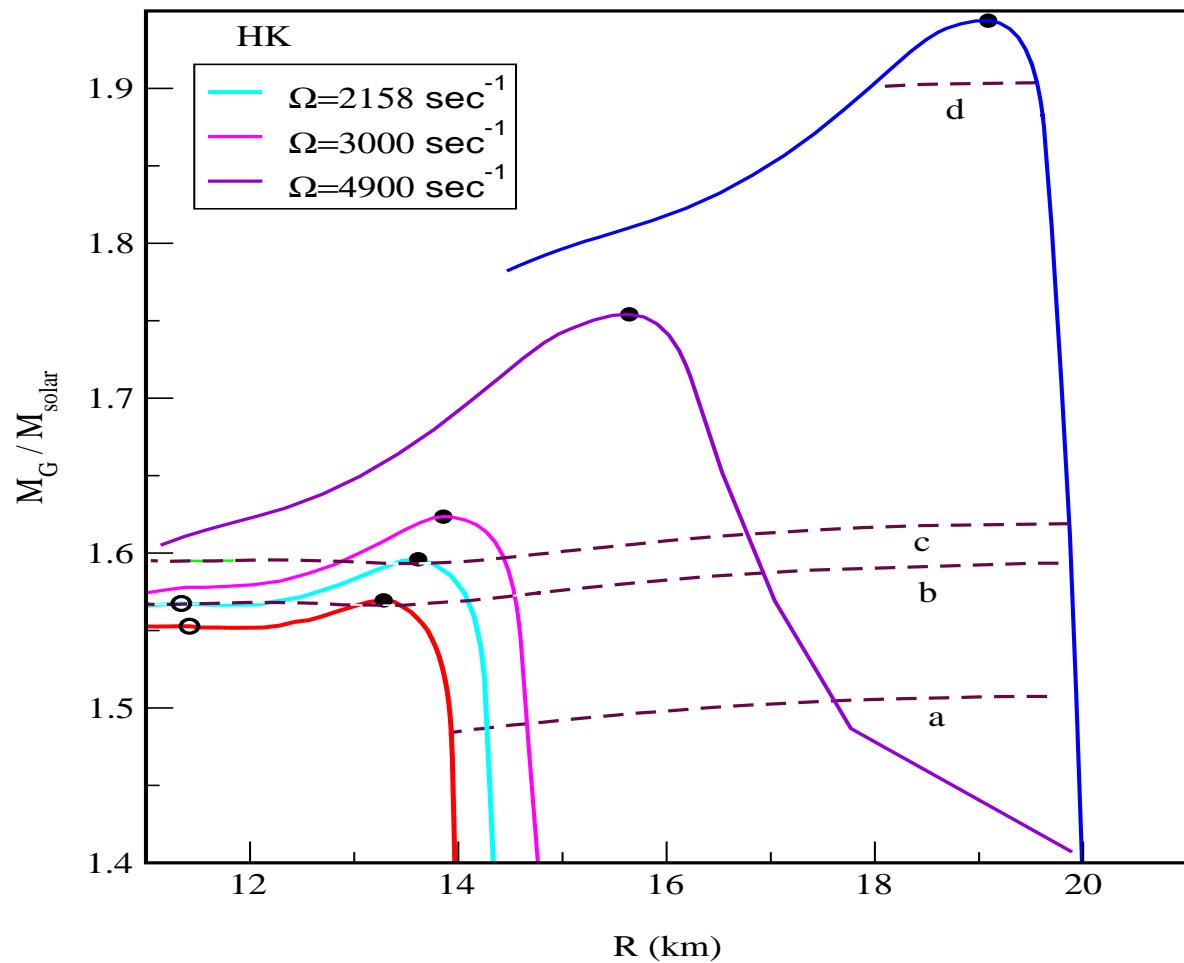
- Metric functions depend on r and θ ,
- Dragging of local inertial frames,
- Centrifugal force stabilises the star against gravitational collapse.

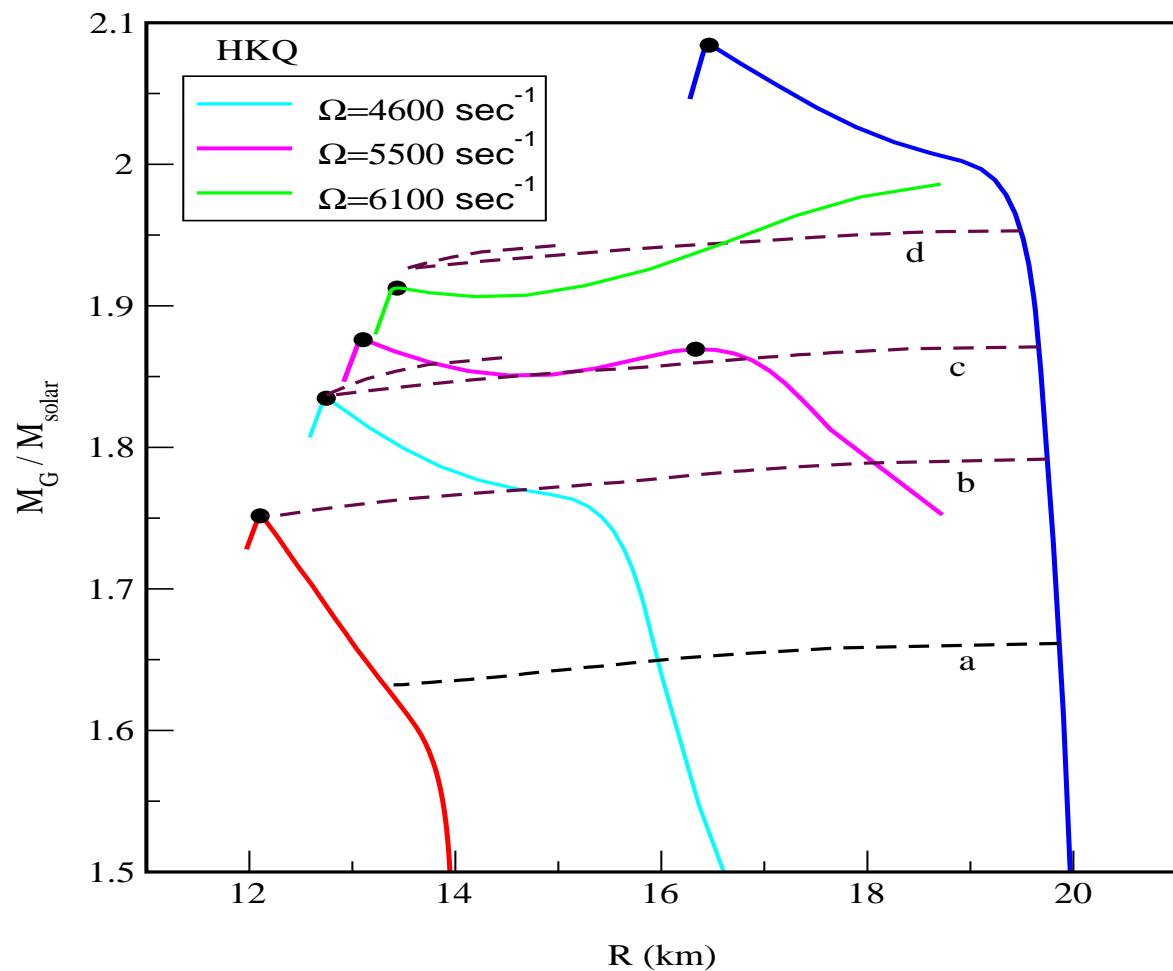
Fixed baryon number evolutionary sequences

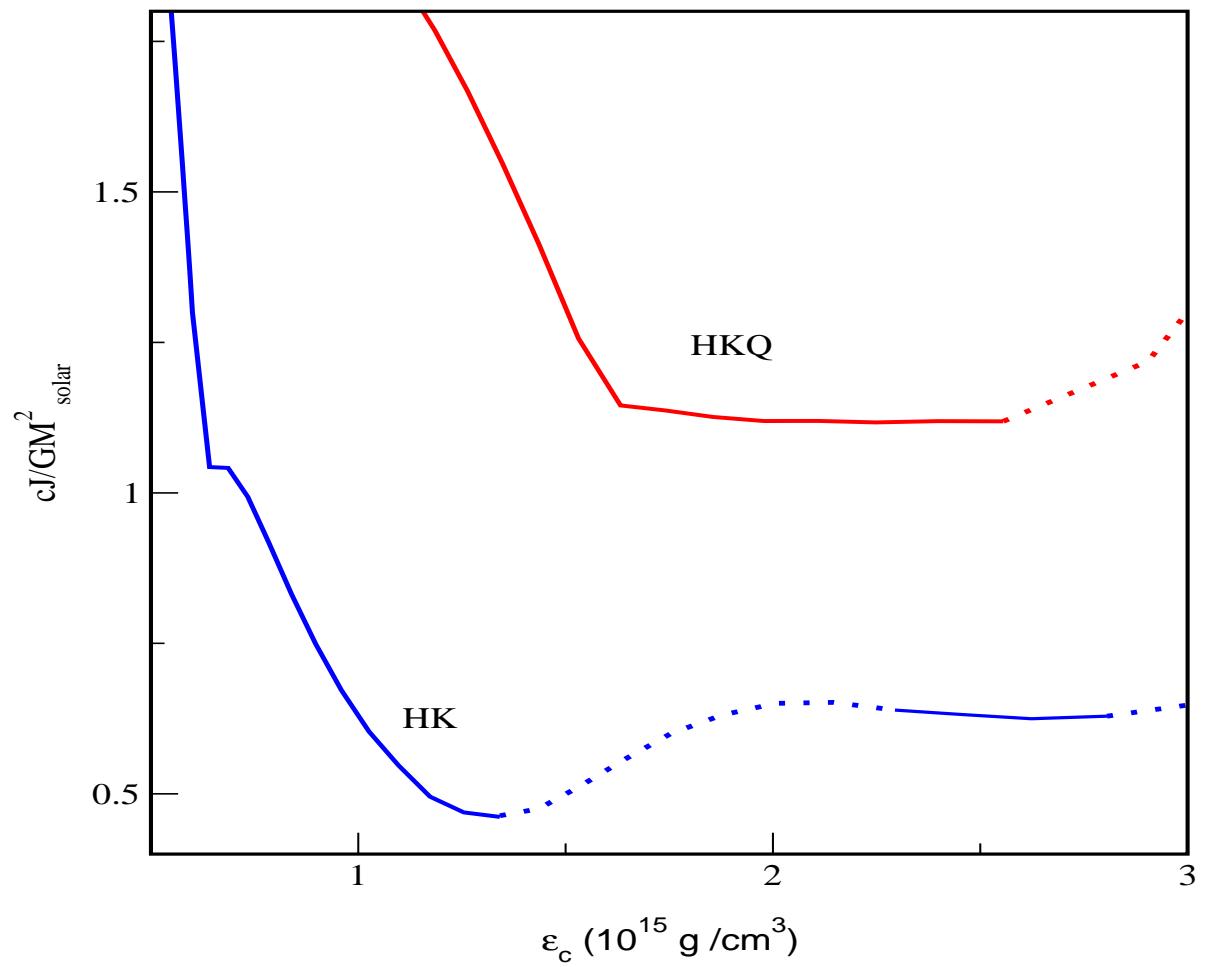
- Normal Sequence
- Supramassive Sequence

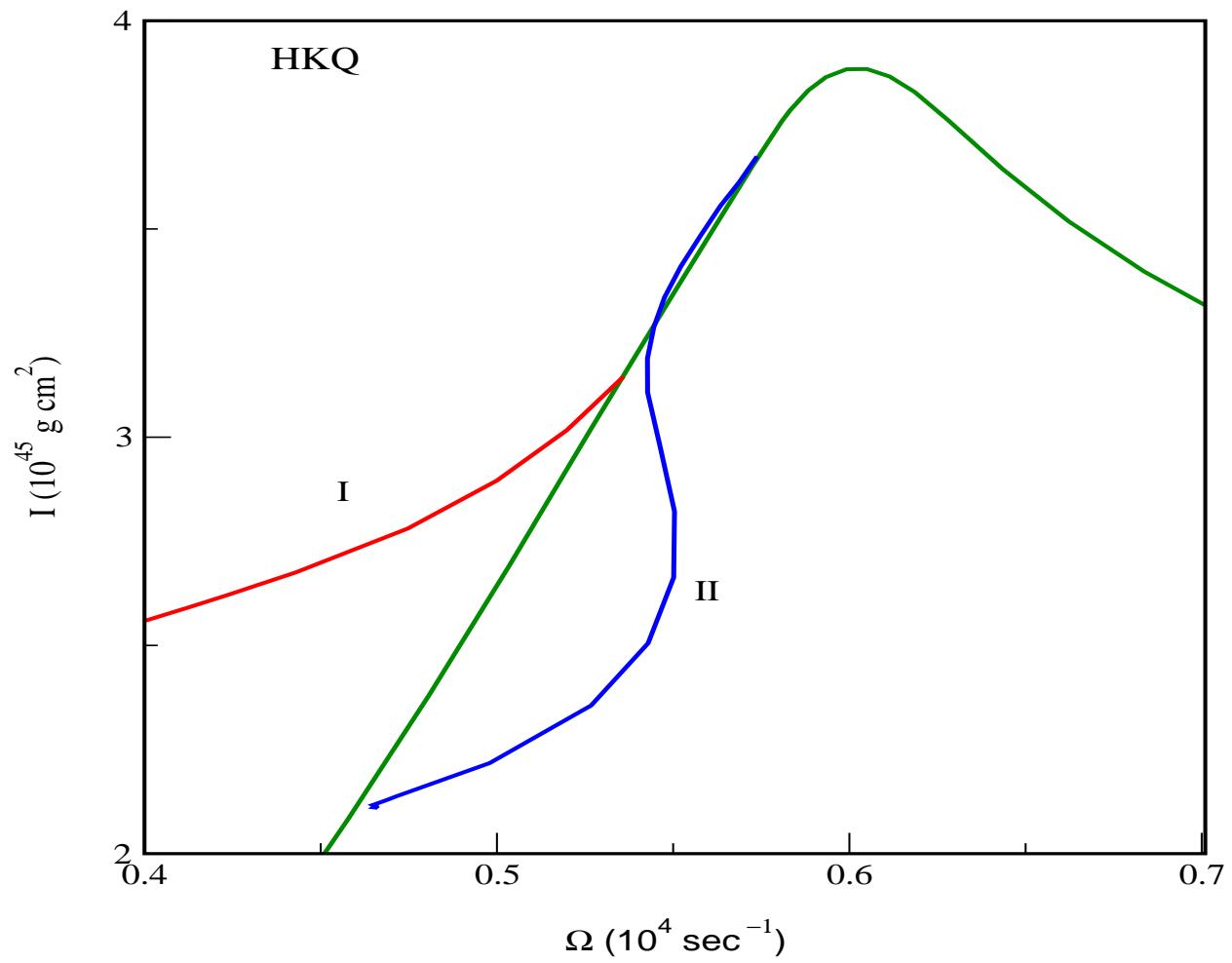
of isolated rotating neutron stars provide informations about phase transitions to Bose-Einstein condensate and quark matter.











Conclusions

The high density behaviour of EoS including exotica in (non)rotating stars exhibits many interesting results.

Back bending phenomenon in rotating stars indicates a strong first order phase transition to quark matter.

Experiments on deeply \bar{K} bound states in nuclei at J-PARC would shed light on the formation of cold and dense matter and put constraint on input parameters such as antikaon optical potential depth.

Our ultimate goal is to constrain the composition and EoS of dense matter in neutron stars and understand the physics of supernovae better.